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"Interactii in teorii cu spin 3/2 si spin 2 mediate de modele topologice de tip BF"

-Rezumatul tezei de doctorat-

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1 Problemele de baza abordate. Metoda de lucru. Ipotezele de lucru

In teza sunt abordate urmatoarele probleme de baza: a) determinarea interactiilor dintre un model BF si campul tensorial cu simetria mixta $(2,1)$; b) constructia cuplajelor dintre un model BF si campul Rarita–Schwinger nemasiv.

Metoda de lucru pe care o vom utiliza se bazeaza pe rezolvarea ecuatiilor care descriu deformarea solutiei ecuatiei master [1] prin intermediul tehnicilor coomologice [2]–[4].

Ipotezele de baza in care construim interactiile mentionate anterior sunt: localitatea spatio-temporala, invarianta Poincare, analiticitatea deformarii in constanta de cuplaj si conservarea numarului de derivate pentru fiecare camp. Primele doua ipoteze implica faptul ca teoria cu interactie trebuie sa fie locala in spatiu-timp si invarianta Poincare. Analiticitatea deformarii se refera la faptul ca solutia deformata a ecuatiei master este analitica in constanta de cuplaj si se reduce la solutia corespunzatoare teoriei libere in limita anularii constantei de cuplaj. Conservarea numarului de derivate pentru fiecare camp implica doua aspecte care trebuie satisfacute simultan: (i) pentru fiecare camp, ordinul ecuatiilor de miscare deduse din teoria libera trebuie sa fie acelasi cu ordinul ecuatiilor de miscare deduse din teoria cu interactie; (ii) numarul maxim de derivate care apar in vertexurile de interactie nu poate sa depaseasca numarul maxim de derivate care apar in Lagrangianul teoriei libere.

2 Rezultate obtinute

In continuare vom prezenta pe scurt rezultatele obtinute.

2.1 Interactiile dintre un model BF si campul tensorial cu simetria mixta $(2,1)$

Punctul de start il reprezinta un model liber in $D = 5$, a carui actiune Lagrangiana se scrie ca suma dintre actiunea pentru un model topologic de tip BF abelian cu un spectru maximal de campuri (un camp scalar φ, doua sorturi de unu-forme H^{μ} si V_{μ}, doua tipuri de doi-forme $B^{\mu\nu}$ si $\phi_{\mu\nu}$ si o trei-forma $K^{\mu\nu\rho}$) si actiunea care descrie un camp tensorial nemasiv cu simetria mixta $(2,1)$ $t_{\mu\nu|\alpha}$ (adica este antisimetric in primii doi indici, $t_{\mu\nu|\alpha}=-t_{\nu\mu|\alpha}$, si satisface identitatea $t_{[\mu\nu|\alpha]}\equiv 0$)

$$\begin{aligned} S_0\left[\Phi^{\alpha_0}\right] = \int d^5x \left(H^{\mu}\partial_{\mu}\varphi + \tfrac{1}{2}B^{\mu\nu}\partial_{[\mu}V_{\nu]} + \tfrac{1}{3}K^{\mu\nu\rho}\partial_{[\mu}\phi_{\nu\rho]}\right. \\ \left. -\tfrac{1}{12}\left(F_{\mu\nu\rho|\alpha}F^{\mu\nu\rho|\alpha} - 3F_{\mu\nu}F^{\mu\nu}\right)\right) \equiv \int d^5x\left(\mathcal{L}_0^{\mathrm{BF}} + \mathcal{L}_0^{\mathrm{t}}\right), \end{aligned} \tag{1}$$

unde am utilizat notatiile

$$\Phi^{\alpha_0} = \left(\varphi, H^{\mu}, V_{\mu}, B^{\mu\nu}, \phi_{\mu\nu}, K^{\mu\nu\rho}, t_{\mu\nu|\alpha}\right), \tag{2}$$

$$F_{\mu\nu\rho|\alpha} = \partial_{[\mu}t_{\nu\rho]|\alpha}, \qquad F_{\mu\nu} = \sigma^{\rho\alpha}F_{\mu\nu\rho|\alpha}. \tag{3}$$

Vom lucra cu metrica Minkowski $\sigma^{\mu\nu}=\sigma_{\mu\nu}=(+----)$.

Actiunea (1) este invarianta fata de setul generator de transformari gauge

$$\delta_{\Omega}\varphi = 0,\ \delta_{\Omega}H^{\mu} = 2\partial_{\nu}\epsilon^{\mu\nu},\ \delta_{\Omega}V_{\mu} = \partial_{\mu}\epsilon,\ \delta_{\Omega}B^{\mu\nu} = -3\partial_{\rho}\epsilon^{\mu\nu\rho}, \tag{4}$$

$$\delta_{\Omega}\phi_{\mu\nu} = \partial_{[\mu}\xi_{\nu]},\ \delta_{\Omega}K^{\mu\nu\rho} = 4\partial_{\lambda}\xi^{\mu\nu\rho\lambda},\ \delta_{\Omega}t_{\mu\nu|\alpha} = \partial_{[\mu}\theta_{\nu]\alpha} + \partial_{[\mu}\chi_{\nu]\alpha} - 2\partial_{\alpha}\chi_{\mu\nu}, \tag{5}$$

in care toti parametrii gauge sunt bosonici, cu $\epsilon^{\mu\nu}$, $\epsilon^{\mu\nu\rho}$, $\xi^{\mu\nu\rho\lambda}$ si $\chi_{\mu\nu}$ complet antisimetrici si $\theta_{\mu\nu}$ simetric. Vom utiliza notatia colectiva Ω^{α_1} pentru toti parametrii gauge

$$\Omega^{\alpha_1} \equiv \left(\epsilon^{\mu\nu}, \epsilon, \epsilon^{\mu\nu\rho}, \xi_{\mu}, \xi^{\mu\nu\rho\lambda}, \theta_{\mu\nu}, \chi_{\mu\nu}\right). \tag{6}$$

Transformarile gauge (4)–(5) sunt abeliene si reductibile de ordinul trei, relatiile de reductibilitate avand loc in spatiul tuturor istoriilor de camp.

Un prim rezultat de baza este dat de urmatoarele doua teoreme.

Teorema 1

A) In ipotezele de lucru mentionate anterior, cea mai generala forma a actiunii cu interactie este data de

$$\begin{aligned}\bar{S}_0\left[\Phi^{\alpha_0}\right] = & \int d^5x\left\{H_\mu\partial^\mu\varphi+\tfrac{1}{2}B^{\mu\nu}\partial_{[\mu}V_{\nu]}+\tfrac{1}{3}K^{\mu\nu\rho}\partial_{[\mu}\phi_{\nu\rho]}\right.\\ & -\tfrac{1}{12}\left(F_{\mu\nu\rho|\alpha}F^{\mu\nu\rho|\alpha}-3F_{\mu\nu}F^{\mu\nu}\right)\\ & +g\left[W_1V_\mu H^\mu+W_2B_{\mu\nu}\phi^{\mu\nu}-W_3\phi_{[\mu\nu}V_{\rho]}K^{\mu\nu\rho}+\bar{M}(\varphi)\right.\\ & \left.+\varepsilon^{\alpha\beta\gamma\delta\varepsilon}\left(9W_4V_\alpha\tilde{K}_{\beta\gamma}\tilde{K}_{\delta\varepsilon}+\tfrac{1}{4}W_5V_\alpha\phi_{\beta\gamma}\phi_{\delta\varepsilon}+W_6B_{\alpha\beta}K_{\gamma\delta\varepsilon}\right)\right]\\ & \left.+g\left(k_1\phi^{\mu\nu}-\tfrac{k_2}{20}\tilde{K}^{\mu\nu}\right)\left[F_{\mu\nu}+\tfrac{3g}{2}\left(k_1\phi_{\mu\nu}-\tfrac{k_2}{20}\tilde{K}_{\mu\nu}\right)\right]\right\}, \end{aligned} \tag{7}$$

unde k_1 si k_2 sunt constante, iar $\bar{M}$, $W_1, \ldots, W_5$ si W_6 sunt functii care depind de campul scalar nederivat. Marimea notata prin g reprezinta constanta de cuplaj (paramatrul de deformare).

B) Constantele k_1 si k_2 si functiile $\bar{M}$, $W_1, \ldots, W_5$ si W_6 satisfac ecuatiile

$$\frac{d\bar{M}\left(\varphi\right)}{d\varphi}W_1\left(\varphi\right)=0, \qquad W_1\left(\varphi\right)W_2\left(\varphi\right)=0, \tag{8}$$

$$W_1\left(\varphi\right)\frac{dW_2\left(\varphi\right)}{d\varphi}-3W_2\left(\varphi\right)W_3\left(\varphi\right)+6W_5\left(\varphi\right)W_6\left(\varphi\right)=0, \tag{9}$$

$$W_2\left(\varphi\right)W_3\left(\varphi\right)+W_5\left(\varphi\right)W_6\left(\varphi\right)=0, \tag{10}$$

$$W_1\left(\varphi\right)\frac{dW_6\left(\varphi\right)}{d\varphi}+3W_3\left(\varphi\right)W_6\left(\varphi\right)-6W_2\left(\varphi\right)W_4\left(\varphi\right)=0, \tag{11}$$

$$W_1\left(\varphi\right)W_6\left(\varphi\right)=0, \qquad W_2\left(\varphi\right)W_4\left(\varphi\right)+W_3\left(\varphi\right)W_6\left(\varphi\right)=0, \tag{12}$$

$$W_2\left(\varphi\right)W_5\left(\varphi\right)=0, \qquad W_4\left(\varphi\right)W_6\left(\varphi\right)=0, \tag{13}$$

$$k_1W_3+\tfrac{k_2}{60}W_5=0, \qquad k_1W_4+\tfrac{k_2}{2\cdot 5!}W_3=0, \qquad k_1W_6+\tfrac{k_2}{5!}W_2=0. \tag{14}$$

C) Ecuatiile (8)–(14) poseda solutii.

Observam ca vertexurile de cuplaj efectiv din (7), si anume

$$g\left(k_1\phi^{\mu\nu}-\tfrac{k_2}{20}\tilde{K}^{\mu\nu}\right)F_{\mu\nu}, \tag{15}$$

sunt de ordinul unu in parametrul de deformare g si cupleaza tensorul $t_{\lambda\mu|\alpha}$ numai cu doi-forma $\phi_{\mu\nu}$ si cu trei-forma $K^{\mu\nu\rho}$ din sectorul BF. Este de asemenea interesant de observat ca aparitia vertexurilor de interactie

$$\tfrac{3g^2}{2}\left(k_1\phi^{\mu\nu}-\tfrac{k_2}{20}\tilde{K}^{\mu\nu}\right)\left(k_1\phi_{\mu\nu}-\tfrac{k_2}{20}\tilde{K}_{\mu\nu}\right) \tag{16}$$

(care descriu selfinteractii in sectorul BF) este strict datorata prezentei tensorului $t_{\lambda\mu|\alpha}$ (in absenta acestuia $k_1 = k_2 = 0$, deci vertexurile de mai sus se anuleaza).

Teorema 2
A) Actiunea (7) este invarianta la transformarile gauge deformate

$$\bar{\delta}_\Omega \varphi = -gW_1\epsilon, \tag{17}$$

$$\begin{aligned}\bar{\delta}_\Omega H^\mu = {} & 2D_\nu \epsilon^{\mu\nu} + g\left(\frac{dW_1}{d\varphi}H^\mu - 3\frac{dW_3}{d\varphi}K^{\mu\nu\rho}\phi_{\nu\rho}\right)\epsilon - 3g\frac{dW_2}{d\varphi}\phi_{\nu\rho}\epsilon^{\mu\nu\rho} \\ & +2g\left(\frac{dW_2}{d\varphi}B^{\mu\nu} - 3\frac{dW_3}{d\varphi}K^{\mu\nu\rho}V_\rho\right)\xi_\nu + 12g\frac{dW_3}{d\varphi}V_\nu\phi_{\rho\lambda}\xi^{\mu\nu\rho\lambda} \\ & +2g\frac{dW_6}{d\varphi}B^{\mu\nu}\varepsilon_{\nu\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta} + 3gK^{\mu\nu\rho}\left(4\frac{dW_4}{d\varphi}V_\nu\varepsilon_{\rho\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta} - \frac{dW_6}{d\varphi}\varepsilon_{\nu\rho\alpha\beta\gamma}\epsilon^{\alpha\beta\gamma}\right) \\ & +g\varepsilon^{\mu\nu\rho\lambda\sigma}\left[\tfrac{1}{4}\frac{dW_4}{d\varphi}\varepsilon_{\nu\rho\alpha\beta\gamma}K^{\alpha\beta\gamma}\varepsilon_{\lambda\sigma\alpha'\beta'\gamma'}K^{\alpha'\beta'\gamma'}\epsilon - \frac{dW_5}{d\varphi}\phi_{\nu\rho}\left(V_\lambda\xi_\sigma - \tfrac{1}{4}\phi_{\lambda\sigma}\epsilon\right)\right],\end{aligned} \tag{18}$$

$$\bar{\delta}_\Omega V_\mu = \partial_\mu \epsilon - 2gW_2\xi_\mu - 2g\varepsilon_{\mu\nu\rho\lambda\sigma}W_6\xi^{\nu\rho\lambda\sigma}, \tag{19}$$

$$\begin{aligned}\bar{\delta}_\Omega B^{\mu\nu} &= -3\partial_\rho\epsilon^{\mu\nu\rho} - 2gW_1\epsilon^{\mu\nu} + 6gW_3\left(2\phi_{\rho\lambda}\xi^{\mu\nu\rho\lambda} + K^{\mu\nu\rho}\xi_\rho\right) \\ &\quad +g\left(12W_4K^{\mu\nu\rho}\varepsilon_{\rho\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta} - W_5\varepsilon^{\mu\nu\rho\lambda\sigma}\phi_{\rho\lambda}\xi_\sigma\right),\end{aligned} \tag{20}$$

$$\begin{aligned}\bar{\delta}_\Omega \phi_{\mu\nu} &= D^{(-)}_{[\mu}\xi_{\nu]} + 3g\left(W_3\phi_{\mu\nu}\epsilon - 2W_4V_{[\mu}\varepsilon_{\nu]\alpha\beta\gamma\delta}\xi^{\alpha\beta\gamma\delta}\right) \\ &\quad +3g\varepsilon_{\mu\nu\rho\lambda\sigma}\left(2W_4K^{\rho\lambda\sigma}\epsilon + W_6\epsilon^{\rho\lambda\sigma} - \tfrac{k_2}{180}\partial^{[\rho}\chi^{\lambda\sigma]}\right),\end{aligned} \tag{21}$$

$$\begin{aligned}\bar{\delta}_\Omega K^{\mu\nu\rho} &= 4D^{(+)}_\lambda\xi^{\mu\nu\rho\lambda} - 3g\left(W_2\epsilon^{\mu\nu\rho} + W_3K^{\mu\nu\rho}\epsilon\right) - 2gk_1\partial^{[\mu}\chi^{\nu\rho]} \\ &\quad -g\varepsilon^{\mu\nu\rho\lambda\sigma}W_5\left(V_\lambda\xi_\sigma - \tfrac{1}{2}\phi_{\lambda\sigma}\epsilon\right),\end{aligned} \tag{22}$$

$$\bar{\delta}_\Omega t_{\mu\nu|\alpha} = \partial_{[\mu}\theta_{\nu]\alpha} + \partial_{[\mu}\chi_{\nu]\alpha} - 2\partial_\alpha\chi_{\mu\nu} + gk_1\sigma_{\alpha[\mu}\xi_{\nu]} - \tfrac{gk_2}{5!}\sigma_{\alpha[\mu}\varepsilon_{\nu]\beta\gamma\delta\varepsilon}\xi^{\beta\gamma\delta\varepsilon}, \tag{23}$$

unde

$$D_\nu = \partial_\nu - g\frac{dW_1}{d\varphi}V_\nu, \qquad D^{(\pm)}_\nu = \partial_\nu \pm 3gW_3V_\nu. \tag{24}$$

B) Algebra transformarilor gauge (17)–(23) este neabeliana si se inchide pe suprafata ecuatiilor de camp care rezulta din (7).
C) Relatiile de reductibilitate corespunzatoare transformarilor gauge (17)–(23) se inchid pe suprafata ecuatiilor de camp care rezulta din (7).

2.2 Cuplajele dintre un model BF si campul Rarita–Schwinger nemasiv

Pornim de la o teorie libera patru-dimensionala a carei actiune Lagrangiana se exprima ca suma dintre actiunea campului Rarita-Schwinger nemasiv si actiunea unei teorii topologice de tip BF cu spectru maximal de campuri (un camp scalar, doua unu-forme si o doi-forma)

$$\begin{aligned} S_0[\psi_\mu, A^\mu, H^\mu, \varphi, B^{\mu\nu}] &= \int d^4x \left(-\frac{\mathrm{i}}{2}\bar{\psi}_\mu \gamma^{\mu\nu\rho}\partial_\nu \psi_\rho + H_\mu \partial^\mu \varphi + \frac{1}{2} B^{\mu\nu}\partial_{[\mu}A_{\nu]} \right) \\ &\equiv \int d^4x \left(\mathcal{L}_0^{\mathrm{RS}} + \mathcal{L}_0^{\mathrm{BF}} \right). \end{aligned} \tag{25}$$

Vom utiliza metrica Minkowski de forma $\sigma^{\mu\nu} = \sigma_{\mu\nu} = (+---)$. Matricile γ satisfac relatiile standard

$$\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2\sigma_{\mu\nu}\mathbf{1}. \tag{26}$$

Vom utiliza conventia ca γ_0 este hermitica si antisimetrica, in timp ce $(\gamma_i)_{i=\overline{1,3}}$ sunt antihermitice si simetrice.

Action (25) este invarianta la transformarile gauge

$$\delta_\epsilon A^\mu = \partial^\mu \epsilon, \qquad \delta_\epsilon H^\mu = 2\partial_\nu \epsilon^{\mu\nu}, \qquad \delta_\epsilon B^{\mu\nu} = -3\partial_\rho \epsilon^{\mu\nu\rho}, \tag{27}$$

$$\delta_\epsilon \varphi = 0, \qquad \delta_\chi \psi_\mu = \partial_\mu \chi, \tag{28}$$

unde parametrii gauge ϵ, $\epsilon^{\mu\nu}$ si $\epsilon^{\mu\nu\rho}$ sunt bosonici, cu $\epsilon^{\mu\nu}$ si $\epsilon^{\mu\nu\rho}$ complet antisimetrici. Parametrul gauge χ este spinor Majorana. Transformarile gauge (27)–(28) sunt abeliene si reductibile de ordinul doi, relatiile de reductibilitate avand loc in spatiul tuturor istoriilor de camp.

Cel de-al doilea rezultat de baza este sintetizat in urmatoarele teoreme.

Teorema 3
In ipotezele de lucru mentionate anterior, cea mai generala forma a actiunii cu interactie este data de

$$\begin{aligned} \tilde{S}_0[A^\mu, H^\mu, \varphi, B^{\mu\nu}, \psi_\mu] = \int d^4x \, [H_\mu \left(\partial^\mu \varphi - gW(\varphi) A^\mu\right) \\ + \frac{1}{2} B^{\mu\nu}\partial_{[\mu}A_{\nu]} - \frac{\mathrm{i}}{2} \left(\bar{\psi}_\mu \gamma^{\mu\nu\rho} \left(\partial_\nu + g\mathrm{i}\gamma_5 A_\nu U_1\right) \psi_\rho \right) \Big], \end{aligned} \tag{29}$$

unde U_1 este o functie arbitrara care depinde de campul scalar nederivat.

Observam ca vertexurile de interactie din (29), si anume

$$\mathrm{i}g\bar{\psi}_\mu \gamma^{\mu\nu\rho}\gamma_5 A_\nu U_1 \psi_\rho, \tag{30}$$

sunt de ordinul unu in parametrul de deformare g.

Teorema 4
A) Actiunea (29) este invarianta la transformarile gauge deformate

$$\bar{\delta}_\epsilon \varphi = gW(\varphi)\epsilon, \tag{31}$$

$$\begin{aligned}\bar{\delta}_{\epsilon,\chi} H^\mu = {} & 2\left(\partial_\nu + g\frac{dW}{d\varphi}A_\nu\right)\epsilon^{\mu\nu} + g\left[\left(\frac{1}{2}\frac{dU_1}{d\varphi}\bar{\psi}_\nu\gamma^{\mu\nu\rho}\gamma_5\psi_\rho\right.\right. \\ & \left.\left. -\frac{dW}{d\varphi}H^\mu\right)\epsilon + \frac{dU_1}{d\varphi}A_\rho\bar{\psi}_\nu\gamma^{\mu\nu\rho}\gamma_5\chi\right],\end{aligned} \tag{32}$$

$$\bar{\delta}_\epsilon A^\mu = \partial^\mu \epsilon, \tag{33}$$

$$\bar{\delta}_{\epsilon,\chi} B^{\mu\nu} = -3\partial_\rho \epsilon^{\mu\nu\rho} + 2gW(\varphi)\epsilon^{\mu\nu} - gU_1(\varphi)\bar{\psi}_\rho\gamma^{\mu\nu\rho}\gamma_5\chi, \tag{34}$$

$$\bar{\delta}_{\epsilon,\chi}\psi_\mu = \partial_\mu\chi - \mathrm{i}gU_1(\varphi)\left(\gamma_5\psi_\mu\epsilon - \gamma_5\chi A_\mu\right). \tag{35}$$

B) Algebra transformarilor gauge (31)–(35) este neabeliana si se inchide pe suprafata ecuatiilor de camp care rezulta din (29).
C) Relatiile de reductibilitate corespunzatoare transformarilor gauge (31)–(35) se inchid pe suprafata ecuatiilor de camp care rezulta din (29).

2.3 Lucrari publicate

Rezultatele de baza ale tezei sunt continute in lucrarile [5]–[10].

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