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Abstract of Ph.D. thesis

Methods and algorithms of nonlinear systems identification

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Craiova 2010

Identification of nonlinear systems is a growing field. Among different approaches on issues such systems were non-parametric methods, which are essentially techniques in the frequency study. These methods include techniques for identification using Volterra series, which characterize the response input - output by an amount of multifrequential convolutions mediation. However, time domain simulation is a convenient construction achievement in nonlinear state-space, which creates difficulties in practice. On the other hand, parametric methods were developed and based on the models in structured or unstructured time. Structural models or black-box is based mostly on neural network approximations in exploration properties functions. Identifying patterns is of considerable importance continued in various fields such as economics, management or automatic signal processing. Thanks to the development of computing and digital data acquisition boards, most identification schemes are focused on identifying the parameters of discrete systems based on input-output sampled. In this sense theories were developed well developed and numerous works have been reported.

The objective of the entire study, materialized in this thesis is focused around the *design, development and implementation of methods and algorithms for the class of nonlinear systems identification*. Following target along the entire sentence was a parallel between theoretical and applied concepts aatfel to obtain a uniform sentences that follow current trends in field identification systems.

The thesis discusses the issue of identifying nonlinear systems along the five chapters addressing issues that arise gradually in the treatment concepts of identification systems

The first chapter presents the history and different approaches to field identification systems. Identification of continuous-time systems makes possible a direct connection to the physical and fundamental operation of systems and a direct estimation of physical parameters that have a clear meaning. Initial framework from which to start the study is provided by determining the concepts that arise when handling a problem of general identification, regardless of type system found. First the whole issue dealt primarily intended subject, which is the system itself. For this reason it appears necessary for full comprehension of the meaning that it becomes subject to the central role in addressing problems of identification. In this first chapter concepts have a logical process, from system definition of all properties related to treatment of certain issues which lead to the development of algorithms for solving the problem of identification. The first chapter's present theoretical concepts are mentioned in the literature, but which leads to focus on main topic, namely the development of algorithms for identifying nonlinear systems. With the establishment of concepts that are working to define problems and to follow a logical, from simple to complex classifications have developed several systems and related models, are presented in this chapter and follow solving such problems of identification.

Chapter two is a natural complement to the framework provided in the first half. As presented in the previous chapter, over time has made some progress in identifying the scope for continuous systems. One of the major trends in treatment systems to identify parameters in achieving a link between the system of differential equations, default physical system and a system of algebraic equations to emulate the original system by focusing dependencies between parameters known to occur and / or of unknown model . Starting from this idea to find a way to provide solutions as precise and as applicable to the development of model systems that certain algorithms can be implemented. Such an approach to problem identification is provided by the application of systems theory distributions aimed at finally getting some algebraic systems of equations describing the fullest physical systems models. Theory of distributions is used in many scientific and engineering applications of which may be mentioned inter aria, the theory of differential equations, quantum mechanics, fluid dynamics, conservation law, etc.

A schematic way of treating a problem of identification of model parameters continuously via an indirect approach is the initial estimate of a discrete model, where it is transformed into an equivalent continuous model. However, getting continuous in the discrete model is not always an easy process because of problems related to sampling period used for data acquisition.

An alternative approach is to achieve a continuous model directly from sampled data input / output. Since the equation error is a function of parameters of linear algebraic model based on equation error methods have been widely used to identify patterns continue, directly from discrete data. The main problems that arise in this approach are the handling and disposal derivatives offset immeasurable asymptotic estimation of parameters where noise is high.

Building on ideas presented in terms of beginning of this chapter and using the properties of distributions, developed a method for identifying both linear systems described by differential equations with constant coefficients, and certain classes of nonlinear systems described by differential equations with coefficients also constant.

As a hypothesis we first considered a linear proper time invariant system, whose order was k with an input generated by $u : R \to \mathrm{R}$ and an output $y : R \to \mathrm{R}$, described through his differential equations model with real coefficients:

$$\sum_{k=0}^{n} a_k y^{(k)}(t) = \sum_{k=0}^{m} b_k u^{(k)}(t),\ m \le n, a_n \ne 0, a_0 = 1 \tag{1}$$

The assumptions applied this model incorporates all the conditions for proper functioning of the system but the parameters and notations. Algorithm for synthesizing first settled derivatives and forms the shape parameter vector input signals respectively output, namely:

$$\theta = [b_m, ..., b_k, ..., b_0, a_n, ..., a_k, ..., a_1]^t = [\theta_1, ..., \theta_p]^t \tag{2}$$

$$y^{(k)} = D^k y,\ k = 0 : n, u^{(k)} = D^k u,\ k = 0 : m \tag{3}$$

The next step was to define a derivation operator to assist in the expression vector system model. Given all the changes and conditions outlined in the opening of this chapter could be reached following axiom: "when referring to a dynamic system S will actually mean parameter vector θ and vice versa. Meaning, there is the following relationship: $S = S(m,n) \Leftrightarrow \theta = \theta(m,n)$ ”

Moving forward, it could summarize the general pattern of an *identification problem* now turned the problem of determining the parameter $\theta = \hat{\theta}$ based on information obtained from the pair of signals input / output (u_T, y_T)

$$\hat{\theta} = \hat{\theta}(u_T, y_T, m, n) \tag{4}$$

So that the derivation operator $q_{\hat{\theta}/(u_T, y_T)}(t) = 0,\ \forall t \in R$ (5)

In the distribution theory the term derivable distribution is introduced like a convention. If $F_q \in \Phi'_n$, then his derivative of k order is a new distribution, $F_q^{(k)} \in \Phi'_n$ defined unique as we can see in the next relation

$$F_q^{(k)}(\phi) = (-1)^k F_q(\phi^{(k)}),\ \forall \phi \in \Phi_n \tag{6}$$

$$\phi \to F_q^{(k)}(\phi) = (-1)^k \int_R q(t)\phi^{(k)}(t)dt \in R \tag{7}$$

We consider that we know the continuous function set of form:

$$u^{(k)} : R \to R, t \to u^{(k)}(t),\ 0 \le k \le m,\ cu\ m \le n, \tag{8}$$

Where $u^{(k)}$ represents the k derivative of the function u(t). With this functions we can generate a set of regulate distributions like the ones below:

$$F_u^{(k)}(\phi) = F_{u^{(k)}} : \Phi_n \to R,\ \phi \to F_{u^{(k)}}(\phi) \quad (9)$$

$$F_{u^{(k)}}(\phi) = \int_R u^{(k)}(t)\phi(t)dt = (-1)^k \int_R u(t)\phi^{(k)}(t)dt \quad (10)$$

It can be writing now a differential equation whose unknown variable it will be a distribution, $F \in \Phi_n'$ of the form:

$$\sum_{k=0}^{n} a_k F^{(k)}(t) = \sum_{k=0}^{m} b_k F_u^{(k)},\ m \le n, a_n \ne 0, a_0 = 1 \quad (11)$$

Where trough $F^{(k)} \in \Phi_n'$ we understand the k order derivative of unknown distribution F, meaning:

$$F^{(k)} : \Phi_n \to R,\ \phi \to F^{(k)}(\phi) = (-1)^k F(\phi^{(k)}),\ \forall \phi \in \Phi_n \quad (12)$$

Furthermore, in the same chapter it was presented a graphical method for representing the relations between elements and finite order distributions. It was considered a general distribution generated by a y function, denoted by F_y. The value of this distribution for a test function φ was given by $F_y(\varphi)$. The derivative of this distribution is F_y' with its value $F_y'(\varphi)$ for the same test function. Since the distribution is a functional, meaning that it is a function of φ variable, witch is a function it self, in Figure 1it was represented their relationship. It can be seen in this figure the relation between the two functions $F_y(\varphi)$ and $F_y'(\varphi)$, for $\varphi = \varphi 1$

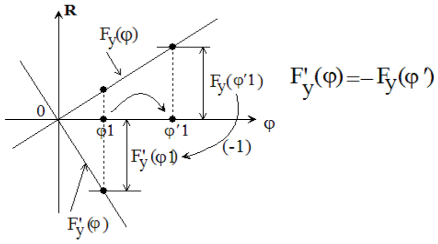


Fig. 1. The relationship between $F_y(\varphi)$ and $F_y'(\varphi)$

Next step was considering the y function witch generates the distribution. The form of this function is given in Figure 2. The three columns of this figure represents: the value of distribution $F_y(\varphi)$, the derivative value of this distribution $F_y'(\varphi)$ and the value $F_{y'}(\varphi)$ generated by $y' = dy/dt$.

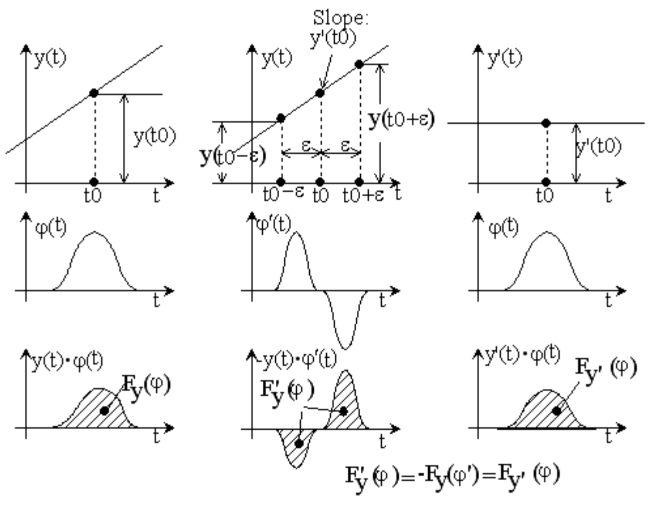


Fig. 2. The link between $F_y(\varphi)$, $F_y'(\varphi)$ and $F_{y'}(\varphi)$

A more general is that the generating function y is locally integrable and has a finite number of discontinuities of the first case. Obviously this function is not differentiable in the classical sense.

The next step in developing the identification algorithm based on the theory of distributions was to attempt to treat simple problems as introduced by using distributions. Solutions have emerged logically from the analysis of all properties present in the theory of functions. Thus, the need to assess the signal derivatives, the integrals of functions that the idea of using elements that can be assessed and a way to force the desired results. These experimental features have shapes who have known characteristics provide a useful tool in evaluating equations describing the system identified.

To illustrate the results obtained by applying identification methods based on distributions was used in the execution of several experiments, one of which will be presented below. Results for a first order variable in time nonlinear system, as described by the model in the next equation where presented

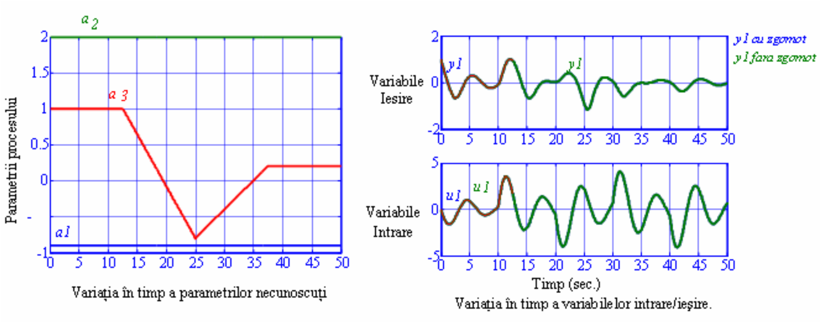
$$\dot{y}(t) = \theta_1 \cdot y(t) + \frac{\theta_3(t) \cdot u(t)}{\theta_2 + y(t)} \tag{13}$$

Where

$$\theta_3(t) = \theta_{31} \cdot t + \theta_{30} \tag{14}$$

The parameters unknown vector has the form:

$$\theta = [\theta_1 \;\; \theta_2 \;\; \theta_{31} \;\; \theta_{30}] \tag{15}$$



Real	**Identified**
2.000	2.00000007054610
-0.900	-0.90000001125488
-0.144	-0.14400000531677
2.800	2.80000010349192

Furthermore, in the **third chapter**, to demonstrate the usefulness of the proposed methods and the ease with which they can develop new algorithms for identifying problems, in addition to the proposed theoretical examples, I devoted a chapter of a larger example and as real. The proposed application seeks to identify the parameters for the complex process of wastewater treatment. Wastewater treatment plants are difficult to control because of nonlinear dynamics and unknown parameters whose values may change over time. Model study on which to work is presented in the following equation:

$$\frac{d}{dt}\begin{bmatrix} X_1 \\ S_1 \\ X_2 \\ S_2 \\ P_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -k_1 & 0 \\ 0 & 1 \\ k_2 & -k_3 \\ 0 & k_4 \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} - D \begin{bmatrix} X_1 \\ S_1 \\ X_2 \\ S_2 \\ P_1 \end{bmatrix} + \begin{bmatrix} 0 \\ DS_{in} \\ 0 \\ 0 \\ -Q_1 \end{bmatrix} \qquad (16)$$

This model describes the behavior of an entire class of bioprocesses, and is known as the general dynamic model of state. Further, they started the scoring parameters and variables involved in the process in characteristic style developed algorithms based on the theory of distribution. Then identification algorithm was applied to different cases of operation of the facility. Results are original and presented in a separate chapter. Continue to refine the algorithm and to increase its accuracy, we proposed a hierarchical structure to identify and estimate such a model equations. Obviously nonlinear identification problem has been preserved in this case. To obtain linear equations in unknown parameters, identification problem is divided into several sub-problems of identification, simple and interdependent layer is called identification. Based on the specific structure of such a system was possible equations of state group, in order to reach the five problems of identification, interconnected and organized in a hierarchical structure. Starting point was the use of several equations of state, the first layer (Layer_a) to obtain a set of linear equations, for some unknown parameters. Identification results of this first stage were then used to express other parameters in linear equations in the second layer and so on. This process is repeated in other layers until all the parameters will be identified. For each layer are affixed same types of procedures and the same numerical algorithm, thus obtaining the general algorithm refinement.

The equations that describes the whole system are given in what it fallows

$$\dot{\xi}_1 = \phi_1 - u_3 \cdot \xi_1 \qquad (17)$$

$$\phi_1 = \theta_5 \cdot \frac{\xi_1 \cdot \xi_2}{\theta_7 + \xi_2} \qquad (18)$$

$$\dot{\xi}_2 = -\theta_1 \cdot \phi_1 - u_3 \cdot \xi_2 + u_1 \cdot u_3 \qquad (19)$$

$$\dot{\xi}_3 = \phi_2 - u_3 \cdot \xi_3 \qquad (20)$$

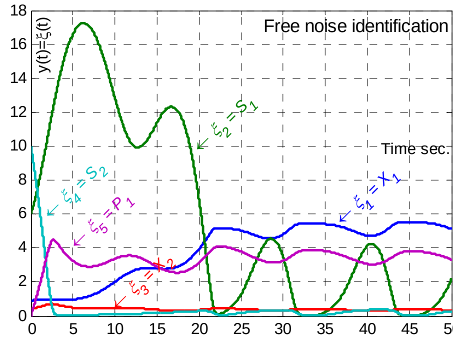
$$\phi_2 = \theta_6 \cdot \frac{\xi_3 \cdot \xi_4}{\theta_8 + \xi_4 + \theta_9^{'} \cdot \xi_4^2} \ , \ \theta_9^{'} = \frac{1}{\theta_9} \qquad (21)$$

$$\dot{\xi}_4 = \theta_2 \cdot \phi_1 - \theta_3 \cdot \phi_2 - u_3 \cdot \xi_4 \qquad (22)$$

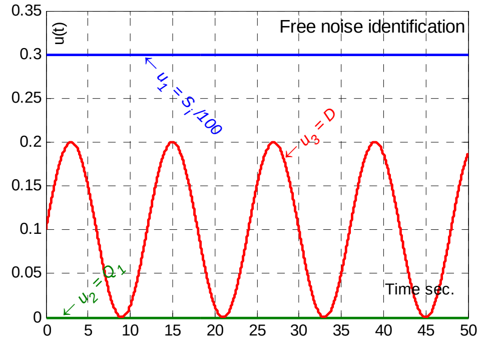
$$\dot{\xi}_5 = -u_3 \cdot \xi_5 + \theta_4 \cdot \phi_2 - u_2 \qquad (23)$$

The results for one of the many experiments are presented in what it fallows

Real Parameters	Identified Parameters
5.40000000000000	5.39999999968776
1.00000000000000	0.99999999998385
14.70000000000000	14.70000000171210
10.00000000000000	9.99999995853515
0.20000000000000	0.20000000003866
0.60000000000000	0.59999999935076
0.75000000000000	0.75000000163917
1.00000000000000	0.99999999888425
21.00000000000000	20.99999976074416



The response of the real system



The evolution of the input-output pair

The hierarchical structure of the algorithm is represented in the next figure

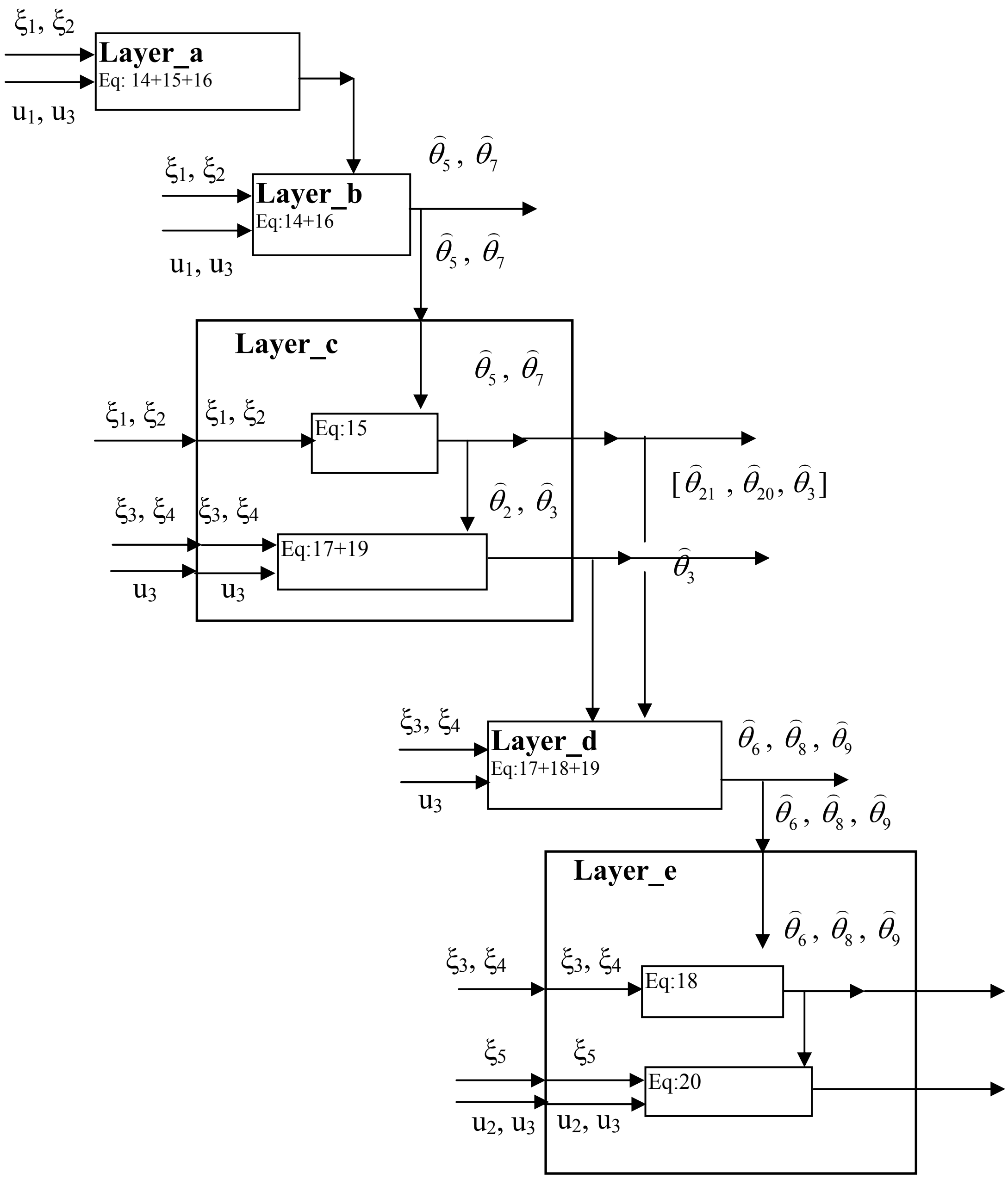


Fig. 3. The hierarchical algorithm of identification

Chapter four is moving on to presenting some of the models for nonlinear systems that have adapted various algorithms to obtain the parameters identified. The most common structure of such systems are nonlinear Hammerstein models, Wiener the systems with nonlinear reaction and combined systems Hammerstein/reaction, whose general form can be seen in Figures 4, 5, 6 and 7. These models imply a linear interconnection between a block and a nonlinear block. Identification of such models has been practiced for a wide range of systems.

Hammerstein model/reaction can be regarded as a realization of a nonlinear system. Consistent representation of a nonlinear system is not unique. Has shown that there is a unique identifying a nonlinear system and changes may be in terms of bias factors has party or nonlinear.

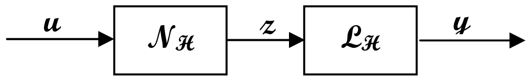


Fig.4. Hammerstein Model

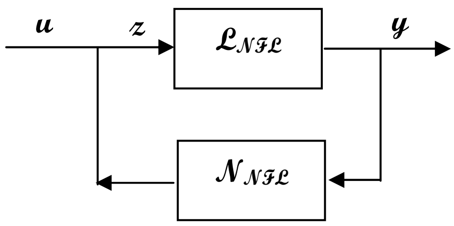


Fig.5. Nonlinear Reaction Model

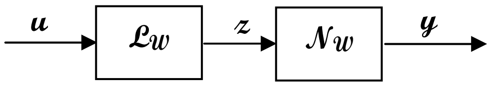


Fig.6. Wiener Model

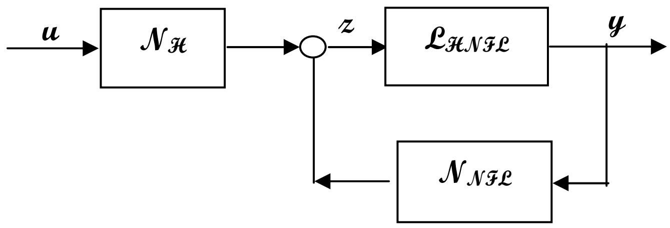


Fig.7. Hammerstein Model with Nonlinear Reaction

To address the identification of parameters for such nonlinear systems in this chapter I've also developed various algorithms that were based on the general theory of identification methods. Thus for case of parameters identification of Hammerstein type nonlinear systems I have started from the general algorithm of least squares by introducing some changes to adapt the method to different cases. After applying the optimization algorithm I passed the results to increase its accuracy. Next, I proposed a more detailed analysis of the basic functions that describe the system by applying various methods of analysis. To highlight the advantages of methods analyzed I've created some applications related to this chapter.

As an example one of the algorithms presented in the thesis is the next experiment. It is considered a system likewise the one in the Fig.6, and whose input-output signals are the ones in the Fig.8. The linear part of this system is given by the transfer function:

$$H(s) = \frac{3s+1}{4s^2 + 0.2s + 1} \quad (24)$$

The nonlinear reaction is given by the function:

$$z(y) = \sum_{j=1}^{6} \gamma_j \cdot B_{j-1}^{5}\left(\frac{y - y_{\min}}{y_{\max} - y_{\min}} \right) \quad (25)$$

Where $B_j^n(s)$ represent the j component of Bernstein function:

$$B_j^n(s) = \frac{n!}{j! \cdot (n-j)!} \cdot s^j \cdot (1-s)^{n-j},\ s \in [0,1] \quad (26)$$

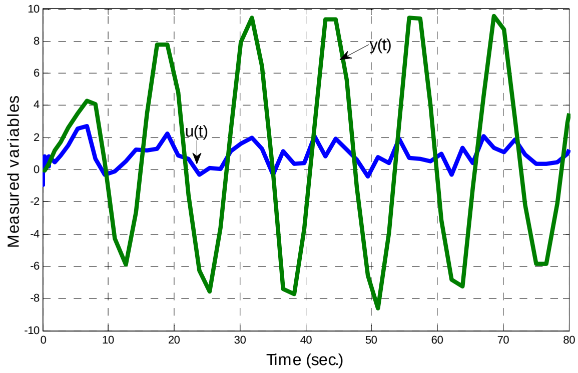


Fig 8. Input/output signals of the identified system

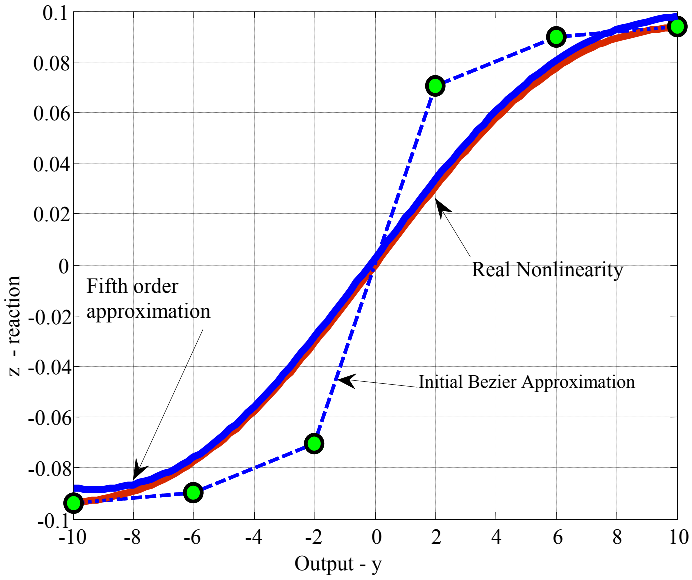


Fig 9. The evolution of the nonlinearities from the process

The dates obtained from one of the experiments:

Parameter	Real value	Identified value
a	$[a_1\ a_2] = [4\ \ 0.2]$	$\hat{a} = [\hat{a}_1\ \hat{a}_2] = [3.678\ \ 0.846]$
b	$[b_1\ b_2] = [3\ \ 1]$	$\hat{b} = [\hat{b}_1\ \hat{b}_2] = [2.878\ \ 0.967]$
γ	$\begin{bmatrix} \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 & \gamma_5 & \gamma_6 \end{bmatrix}$ $\begin{bmatrix} -0.0937 & -0.0895 & -0.0705 & 0.0705 & 0.0895 & 0.0937 \end{bmatrix}$	$\begin{bmatrix} \hat{\gamma}_1 & \hat{\gamma}_2 & \hat{\gamma}_3 & \hat{\gamma}_4 & \hat{\gamma}_5 & \hat{\gamma}_6 \end{bmatrix}$ $\begin{bmatrix} -0.0881 & -0.0935 & -0.0653 & 0.0731 & 0.0913 & 0.0987 \end{bmatrix}$

Chapter five raised as a natural extension of concepts presented in previous chapters. To demonstrate the applicability and utility of systems analysis methods and algorithms I have developed, furthermore I have implemented on a physical installation. Some algorithms were used to identify parameters of temperature control equipment. For testing I used an experimental platform for climate control.

Personal contributions resulting from whole this research are mainly concentrated in the **last chapter**, but they are also outlined in previous chapters. In this doctoral thesis I developed a series of algorithms and identification methods for nonlinear continuous systems. Next I briefly expose the main personal and original contributions of this thesis:

1. Improvement of the nonlinear systems identification problem using distributions. Using this method can identify the various parameters either linear or nonlinear systems without the need of a priori information about system structure. Information obtained by using the distributions is not based on some special values of system response to a particular point in time domain.
2. Design, development and implementation of algorithms for nonlinear systems identification using distributions. The main advantages of this method represents the restrictions that put aside the effect of variables such systems should not prevent assessment estimated derivatives of signals that induce nonlinearities.
3. Adaptation and implementation of software test types of functions used to transform differential equations into algebraic equations with respect to unknown parameters of the system.
4. Synthesis algorithms for nonlinear systems identification Hammerstein, Wiener respectively.
5. From physical demonstration I have tested the methods on an experimental installation of temperature control of a chamber (HVAC Workbench ELVIS platform, offered by National Instruments).
6. Developing a MATLAB software package for systems identification algorithms, as follows: identification programs using distributions functions, adapting programs to implement test functions and their derivatives, nonlinear systems identification programs continue using distributions, nonlinear systems identification programs continuous type characterized by polynomial equations, using Hammerstein and Wiener distributions, program identification of classes of distributions using biotechnological processes, data acquisition software using LabView software, definition and construction, as Matlab procedures, test functions of a key area.

This paper address current trends in nonlinear systems identification required making a contribution in the development of methods for identification but also in designing certain algorithms that match the needs of implementing.

The study developed in this thesis has highlighted the importance of identifying systems using different methods so that the reaction systems after application identification algorithms to maintain. Elements, concepts and structures developed and tested in this thesis identify underlying modeling and providing an alternative to study both theoretically and practically.