UNIVERSITY OF CRAIOVA
FACULTY OF PHYSICS

EUGEN DIACONU

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COOMOLOGICAL ASPECTS OF N=1, D=11 SUGRA

Ph.D. supervisor
Prof. dr. CONSTANTIN BIZDADEA

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Contents

SUMMARY

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Since its appearance in the early seventies, supersymmetry has become a central issue in the quest for unifcation of the fundamental forces of Nature. Supersymmetry is a model which unifies fermions (matter) with bosons (carrier off forces), either in flat space (supersymmetry) or in curved space-time (supergravity). Every fermion would have a bosonic counterpart, and vice versa. Supersymmetric models (SUSY) are described by graded Lie algebras with commuting and anticommuting parameters. In addition to the generators of Lorentz transformations and translations in a D-dimensional space-time, the supersymmetry algebra contains one or more spinor supercharges ('simple'or 'N-extended' supersymmetry). This type of supersymetric algebras was first introduced in the context of the string models to unify the bosonic and the fermionic sectors and in elementary particle physics. It was later shown by Wess and Zumino how to construct a $3+1$ dimensional field theory which was invariant under this symmetry. Despite the fact that none of the supersymmetric models has been signaled yet by experiments, such theories still remain of present interest. The main reason is that radiative corrections tend to be less important in supersymmetric theories, due to cancellations between fermion loops and boson loops.

In supersymmetric theories, Einstein's general theory of relativity turns out to be a necessary consequence of a local gauged SUSY. Thus, local SUSY theories provide a natural framework for the unification of gravity with the other fundamental interactions of nature. On a more fundamental level, in the context of recent developments in string/brane theory, supergravity (SUGRA) in 11 dimensions seems to play an important role. Such a string, or membrane theory is expected to manifest itself in a four-dimensional point particle limit as some locally supersymmetric effective theory. Moreover, it was shown by Scherk that $N = 1$, $D = 11$, SUGRA yields the $N = 8$, $D = 4$ model upon dimensional reduction. Of the many special properties of $D = 11$, $N = 1$ supergravity, one of the most striking is that it forbids a cosmological term. The proof of this result has been done using a combined technique — the standard Noether current method and a cohomological approach.

It is known that the field content of $D = 11$, $N = 1$ supergravity is quite simple; it comprises a graviton, a massless Majorana spin-3/2 field, and a three-form gauge field. The analysis of all possible interactions in $D = 11$ related to this field content necessitates the study of cross-couplings involving each pair of these sorts of fields and then the construction of simultaneous interactions among all the three types of fields. One of the most efficient and meanwhile elegant approaches to the problem of constructing consistent interactions in gauge field theories is that based on the deformation technique combined with local BRST cohomology. This approach relies on computing the deformations of the solution to the master equation for the interacting theory with the help of the 'free' BRST cohomology.

The main goal of this paper is to construct all consistent interactions in $D = 11$ that can be added to a free theory describing a Pauli-Fierz graviton, a massless Rarita-Schwinger gravitino, and an Abelian three-form gauge field from the deformation of the 'free' solution to the master equation. By 'consistent' we mean that the interacting theory preserves both the field content and the number of independent gauge symmetries of the free one. The analysis is performed such that the interactions satisfy some general and quite natural

assumptions: smoothness in the coupling constant, locality, Lorentz covariance, Poincaré invariance, and preservation of the number of derivatives on each field (derivative order assumption). Poincaré invariance means that we do not allow explicit appearance of the space-time coordinates in the interacting vertices. The derivative order assumption implies two conditions to be simultaneously satisfied: i) the preservation of the differential order of the free field equations at the level of the coupled theory, and ii) the presence of at most two derivatives in the Lagrangian of the interacting theory (the same number of derivatives like in the free Lagrangian). One of the final outcomes of this procedure will be the quest for the uniqueness of $D = 11$, $N = 1$ SUGRA.

The main results developed in this work can be synthesized into

- the derivation of consistent interactions among an Abelian three-form gauge field and a Pauli-Fierz graviton;
- the investigation of the consistent couplings between an Abelian three-form gauge field and a massless spin-3/2 field;
- the analysis of the cross-couplings among a Pauli-Fierz graviton and a massless Rarita-Schwinger field;
- the determination of all possible interactions among a graviton, a massless spin-3/2 field, and a three-form gauge field;
- the absence of consistent interactions at order one in the coupling constant in the simultaneous presence of all the three types of fields;
- the cohomological proof of the uniqueness of $D = 11$, $N = 1$ SUGRA.

These results are contained in our papers [2]-[11].

The present work is organized into seven chapters.

In chapter 2 we briefly approach the problem of constructing consistent interactions in gauge theories, in the framework of the BRST formalism. It is well-known that the canonical action of the BRST symmetry s is generated by the solution to the master equation (at Lagrangian level) or by the BRST charge (in the Hamiltonian approach). Due to the fact that the solution to the master equation captures all the information for a gauge theory, the problem of constructing consistent interactions in such theories can be reformulated as a problem of deformation of the solution to the master equation associated to the 'free' model. Futhermore, this deformation of the solution to the master equation induces de deformation of both BRST charge and BRST-invariant Hamiltonian. As an example, we present the case of Hamiltonian Stueckelberg coupling between p- and $(p+1)$-form gauge fields. This chapter exhibits a mostly monographic character, the genuine contribution of the author being contained in the paper [1].

Chapter 3 is devoted to the the derivation of cross-interactions between an Abelian three-form gauge field and a Pauli-Fierz graviton in eleven-dimensional Minkowski space-time. We employ the method of constructing consistent interactions based on the deformation of the solution to the master equation associated to the 'free' theory. The starting model is invariant under an Abelian and second-stage 'off-shell' reducible set of gauge symmetries. Making use of the vielbein formulation of spin-two field theory, it turns out that the interacting Lagrangian follows the General Relativity prescriptions and the gauge algebra remains 'off-shell' second-stage reducible. Consequently, we show the uniqueness of interactions. The results presented in this chapter are contained in our papers [2], [6]-[7], [9]-[11].

In chapter 4 we develop the second step of our approach and analyze the consistent eleven-dimensional interactions that can be introduced between a massless Rarita-Schwinger field and an Abelian three-form gauge field. The gauge algebra of the 'free' model is Abelian and 'off-shell' second-stage reducible. Our main result is that under the working hypotheses there are neither cross-couplings nor self-interactions for the gravitino in $D = 11$. The only possible term that can be added to the deformed solution to the master equation is nothing but a generalized Chern-Simons term for the three-form gauge field, which brings contributions to the deformed Lagrangian, but does not modify the original, Abelian gauge transformations. Our result does not contradict the presence in the Lagrangian of $D = 11$, $N = 1$ SUGRA of a quartic vertex expressing self-interactions among the gravitini. We will see in the next chapter that this vertex, which appears at order two in the coupling constant, is due to the *simultaneous* presence of gravitini, three-form, *and* graviton. The content of this chapter is included into our papers [3], [8]-[11].

Chapters 5 follows closely the general line of the previous two. Here we analyze the consistent couplings that can be introduced between a massless spin-two field (described in the free limit by the Pauli-Fierz action) and a massless Rarita-Schwinger spinor in eleven space-time dimensions. The 'free' model is invariant under an irreducible and Abelian generating set of gauge transformations. Under the working assumptions, we prove that in $D = 11$ there are no cross-interactions between the graviton and the massless gravitini and also no self-interactions among the gravitini. Furthermore, we comment on the absence of self-interactions among the gravitini in $D = 11$ and argue that this result does not contradict the presence in the Lagrangian of $D = 11$, $N = 1$ SUGRA of a quartic gravitini vertex. We also make the comparison with the case $D = 4$, where gravitini are known to allow self-interactions in the presence of a graviton, such that their 'mass' constant becomes related to the cosmological one. This chapter is based on our papers [4], [9]-[11].

In chapter 6 we use the results from the previous chapters and approach the fourth (and final) step of constructing all possible interactions in $D = 11$ among a graviton, a massless Majorana spin-$3/2$ field, and a three-form gauge field. First, we put all the fields together and investigate if there are consistent interactions vertices *at order one* in the coupling constant involving *all* of them. The answer is negative, such that the first-order deformation of the solution to the master equation is completely known from the previous steps. Second, we analyze the consistency of the first-order deformation at order two in the coupling constant. This restricts the six constants that parameterize the first-order deformation to satisfy a simple, algebraic system. There are two types of solutions, but only one is interesting from the point of view of interactions (the other allows at most the interactions between a graviton and a 3-form). Third, we analyze this solution and observe that it systematically reproduces the Lagrangian formulation of $D = 11$, $N = 1$ SUGRA. Therefore, we can state that all consistent interactions in $D = 11$ among a spin-2 field, a massless Majorana spin-$3/2$ field, and a three-form that comply with our working hypotheses are uniquely described by $D = 11$, $N = 1$ SUGRA. The analysis performed here is based on our papers [5], [9]-[11].

The last chapter exposes the main conclusions of the present thesis.

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