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“Alternative approaches to gravity theory”

–Ph.D. thesis abstract–

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1 Main subjects approached in the thesis. Method and working hypotheses

Topological BF-type models are important in view of the fact that certain non-Abelian, interacting versions are correlated to an algebraic, Poisson-like structure present in various Poisson sigma models, which are known to describe two-dimensional gravity theories. It is also known that pure gravity in three dimensions is in fact a BF theory. Moreover, General Relativity and Ashtekar formalism in higher dimensions can also be reformulated as topological BF theories in the presence of additional constraints. On the other hand, tensor fields with mixed symmetries are involved in many important theories from Physics, such as superstrings, gravity models, or higher spin supersymmetric theories. The study of gauge theories based on tensor fields with mixed symmetries solved many interesting issues, like, for instance the dual formulation of spin-two or higher theories, the impossibility of consistent interactions within the dual formulation of linearized gravity (DFLG), or the effective construction of certain interactions in gravity-like models.

The above considerations motivate the main problems approached in this thesis, namely: a) construction of self-interactions that can be added to a topological BF model with a maximal field spectrum in $D = 6, 7, 8$; b) generalization of the above results to an arbitrary space-time dimension $D > 8$; c) computation of interactions in $D = 6$, $D = 7$, and $D = 8$ between a topological BF model with a maximal field spectrum and the DFLG in terms of a massless tensor field with the mixed symmetry $(3, 1)$, $(4, 1)$, and respectively $(5, 1)$; d) generalization of the basic results from the above step to the consistent couplings between a BF model and the DFLG in terms of a massless tensor field with the mixed symmetry $(k, 1)$ in an arbitrary dimension $D = k + 3 > 8$. This topic is approached within the general framework of the method of constructing consistent interactions in gauge field theories based on the deformation of the canonical generator of the Becchi-Rouet-Stora-Tyutin (BRST) symmetry, also known as the solution to the classical master equation. [1]. The mathematical tools used at a large extent require various computations of cohomological algebras, including local ones, associated with the accompanying differentials [2]–[4]. The resulting deformations are selected according to the following general hypotheses: analyticity in the coupling constant, space-time locality, Lorentz covariance, Poincaré invariance, and the conservation of the number of space-time derivatives with respect to each field. Since the free field equations of a BF model with a maximal field spectrum are first-order in the derivatives and those corresponding to the DFLG are second-order, we will ensure a unitary background by asking that the maximum derivative order of the Lagrangian density describing the self-interacting BF model, as well as the "effective" couplings between the two theories, is at most equal to one (the derivative-order assumption).

The present work in organized into seven chapters, five appendices and references.

All the results presented below in brief are published in the papers [5]–[15].

2 Results

Initially, we present the results regarding the topological BF model with a maximal field spectrum in an arbitrary dimension D. During the first step of the deformation procedure we construct the first-order deformation (with respect to the coupling constant λ) of the solution to the master equation in agreement with the above mentioned hypotheses and show that its non-integrated density satisfies the following properties:

1.1 its development according to the antighost number (agh) stops at the value D equal to the Minkowski space-time dimension where the theory evolves;

1.2 all the terms of maximum antighost number produce, via the consistency equations specific to the first-order deformation, non-trivial and consistent pieces;

1.3 all the potentially independent consistent solutions to the homogeneous equations of strictly positive values of agh that must be added in each step while solving these equations may be eliminated in the sense that they are either inconsistent or produce terms that can be absorbed within those originating from the components of maximum antighost number;

1.4 the independently consistent solution to the "homogeneous" equation in antighost number 0 can no longer be eliminated and reduced to an arbitrary (at this stage) function on the undifferentiated scalar field, denoted by M;

1.5 the maximum derivative order assumption is required only at the computation of the "homogeneous" solution from the above item, unlike the other working hypotheses, which are needed more frequently.

The expressions of the terms of maximum antighost number follow from coupling the elements of the basis in the γ-invariant BF ghosts of pure ghost number D, $e^{D}(\mathrm{BF})$, to the generic representative from the local cohomology of the Koszul–Tate differential at antighost number D computed in the algebra of invariant "polynomials", $H_{D}^{\mathrm{inv,BF}}(\delta|d)$. There are two types of γ-closed ghosts. The former, denoted by η, corresponds to the lower range of values with respect to the pure ghost number and is associated with the gauge symmetries of the A forms of strictly positive form degree present in the field spectrum. The latter, denoted by C, is associated with the complementary range, of higher values of the same degree, and is due to the gauge invariances of the forms B. The structure of the terms of maximum antighost number, D, present in the first-order deformation is characterized by the following aspects:

a. all terms containing products among three or more ghosts C and respectively those without such ghosts, but involving less than three η's are excluded;

b. the elements of the basis $e^{D}(\mathrm{BF})$ containing at most one C ghost can be represented by means of all distinct solutions to two different classes of Diophantine equations complemented by precise selection rules. Both classes count the number N of ghosts η allowed in each element of this basis. The first class allows precisely one C-type ghost and also at least one or, depending on the situation, two η-like ghosts, while the second class contains only η-like BRST generators (at least three);

c. unless the elements from item a, in special dimensions there appear three more representatives, all involving exactly two C's, namely, one in $D = 2m + 1$, the other in $D = 4m$, and another in $D = 4m + 1$ (the last also contains an η-like ghost);

d. the non-trivial representative of the space $H_D^{\mathrm{inv,BF}}(\delta|d)$ exhibits a polynomial structure with respect to the set of antifields related to the form field $\overset{[1]}{B}$ and its associated C ghosts and is constructed from the first- and higher-order derivatives (of order D inclusively) of a smooth, arbitrary function of the undifferentiated scalar field;

e. in order to "glue" properly the elements of the basis from items a. and c. to the generic representative mentioned previously, we add one distinct function of the undifferentiated scalar field, denoted by Z or W, for each element. Wherever this applies, we index these functions according to the independent solutions of the associated Diophantine equations plus accompanying selection rules.

Starting from the pieces of maximum antighost number, D, we generate all the other components from the first-order deformation of decreasing values of the antighost number up to 0 inclusively. We notice that in strictly positive values of agh there appear more and more frequently terms that can be organized precisely via some (non-trivial) representatives of $H^{\mathrm{BF}}(\delta|d) \setminus H^{\mathrm{inv,BF}}(\delta|d)$ (meaning they are not γ-invariant). In view of the more and more complex, tree-like structure of these representatives originating from pieces of antighost number associated with higher values of N, it is natural to express them in a compact form by means of some suggestive notations. If we re-organize the pieces of the first-order deformations with various values of the antighost number according to the distinct functions of the type Z or W (completely arbitrary at this stage), then several important properties can be shown to hold, namely: each is non-trivial, decomposes into terms of antighost number ranging between 0 and D, satisfies separately the first-order deformation equation, and complies with all the working hypotheses. The same observations remain true at the level of the independently consistent solution of the "homogeneous" solution in antighost number 0, M. Since they underlie independently consistent components of the first-order deformations, we will call Z, W, and M parameterizing functions. With the first-order deformation at hand, next we solve the equation that governs the second-order (in λ) deformation of the solution to the master equation and observe that it holds iff the parameterizing functions are no longer arbitrary, but satisfy a set of algebraic and differential equations, to be called consistency equations. Although it is virtually impossible to identify the concrete form of all these equations in an arbitrary dimension D, nevertheless we are able to display some of them, along with their general structure, expressed as a sum of products between exactly two parameterizing functions or between one function and the first-order derivative (obviously with respect to the scalar field) of another. This peculiar structure allows one to emphasize at least one non-zero solution of the consistency equations, which is extremely important due to the fact that both the lack of solutions as well as the existence of only purely trivial solutions would forbid the construction of self-interactions. Under these circumstances, it is shown that the deformations of order two and higher can be taken to vanish on their solutions. Consequently, the complete deformation of the solution to the master equations, non-trivial and consistent to all orders in the coupling constant, which, moreover fulfills all the working hypotheses, actually reduces to the sum between the solution to the master equation for the free limit of the BF model and the first-order deformation. By means of projecting the overall deformation on various values of the antighost number we deduce the full Lagrangian formulation of the self-interacting BF model in an arbitrary dimension D. The main aspects emerging from this procedure are emphasized below:

i. the entire deformed Lagrangian density is of order one in λ and, very important, there appears a one-to-one correspondence among the self-interaction vertices involving forms denoted by A or B of strictly positive form degree and the terms of maximum antighost number listed at items a.–e. via the identifications $C \longleftrightarrow B$ and $\eta \longleftrightarrow A$;

ii. more precisely, vertices with more than two B-like forms as well as those with no B forms, but comprising less than three A fields, are forbidden;

iii. various classes of couplings with a single B field and at least one or respectively two A forms (all these preserve the PT invariance), and respectively with minimum three A fields, but with no B's (the last ones break the PT symmetry) are allowed. These classes are precisely associated with each distinct solution of the two classes of Diophantine equations introduced at item a. (in this context N counts the number of A-type fields from each vertex);

iv. in the special dimensions mentioned at item c. in the above there appear three more vertices with exactly two B's, namely one in $D = 4m$, another in $D = 2m + 1$, and the third in $D = 4m + 1$ (the last contains also an A form);

v. all the couplings discussed so far depend on the associated functions Z or W and, in addition, the function denoted by M is also present. These functions are no longer arbitrary, but non-zero solutions to the consistency equations;

vi. in principle, the gauge transformations of all fields from the maximal field spectrum get modified with respect to the free limit by terms of order one in λ. A novel feature is revealed in respect with the scalar field (gauge-invariant in the free limit), which gains now non-trivial gauge symmetries;

vii. the entire gauge structure of the interacting theory at order one in λ is reshaped compared with the original one — the gauge algebra becomes open and the reducibility relations no longer hold in the space of all field histories, but only on the deformed stationary surface, while preserving the initial number of independent gauge symmetries from the generating set.

Next, we synthesize the results on the consistent couplings between a topological BF model with a maximal field spectrum and the DFLG in terms of a tensor field with the mixed symmetry $(k, 1)$ in an arbitrary dimension $D = k + 3$. The first-order deformation decomposes as a sum between the component involving only the BRST generators from the BF sector and the piece that effectively couples at least one BRST generator from each sector (BF and respectively $(k, 1)$) [in $D = k + 3$ the DFLG under consideration allows no non-trivial self-interactions]. The former component displays precisely the structure discussed in the above adapted to $D = k + 3$. With respect to it we maintain the notations M, Z and W at the level of the parameterizing functions. The first-order deformation associated with the "effective" couplings requires a more elaborate analysis. The general cohomological properties of the two free theories under study allow terms of maximum antighost number $k+3$. Nevertheless, they do not comply with the general working hypotheses. The next possible terms, of maximum antighost number $k+2$, are shown to be inconsistent and therefore must be suppressed. The same observation is partially valid in relation with the following terms, of maximum antighost number $k+1$. Moreover, none among the potentially independently consistent homogeneous solutions of agh between k and 0 contributes to the first-order deformation (they are either inconsistent, or can be absorbed within the terms of lower agh originating, via the consistency equations satisfied by the components of the first-order deformation, from the terms of maximum antighost number $k+1$, or are consistent, but trivial). As a consequence, the consistent and non-trivial first-order deformation, which effectively couples the two theories and satisfies the working hypotheses can be developed according to agh into pieces between 0 and $k+1$. Its structure exhibits several resemblances to that describing the BF self-interactions. Related to its component of maximum antighost number, $k+1$, we notice that:

2.1 all terms are linear in the antisymmetric first-order derivatives of the pure ghost number 1 ghost field from the $(k,1)$ sector, denoted by $\mathcal{F}$;

2.2 in connection with the BF ghost field content, there appear three types of elements, namely, the first linear in the C ghost of pure ghost number (pgh) equal to k associated with the gauge symmetries of the B form of degree 3 (without other ghosts), the second containing exactly one C ghost (different from the previous one) and meanwhile at least one η ghost, and the third constructed only out of the η ghosts (at least two);

2.3 the last two objects from the previous item are nothing but elements of the basis of pgh k from the BF sector including at most one C ghost and they may be represented again via all the distinct solutions to two classes of Diophantine equations in the presence of well-defined selection rules (different from the two classes required at the description of the first-order deformation from the BF sector in $D=k+3$);

2.4 for both classes a prominent role is played by the number N that counts the ghosts denoted by η;

2.5 all the elements mentioned so far depending on the ghost fields must be coupled only to the generic representative of agh $k+1$ from $H_{k+1}^{\mathrm{inv,BF}}(\delta|d)$ with a polynomial structure in terms of the antifields corresponding to the field $\overset{[1]}{B}$ and its C ghosts and depending on the derivatives of a smooth, arbitrary function on the undifferentiated scalar field;

2.6 the connection of this representative to the product between $\mathcal{F}$ and each element from item 2.2 is accomplished also through functions denoted in this case by V or U, indexed in terms of the distinct solutions to the Diophantine equations (wherever this applies).

In what follows we proceed along the same line applied to the pure BF model and generate all the remaining components of the first-order deformation, of agh between k and 0. Each depends linearly of a single BRST generator from the $(k,1)$ sector, which may be the ghost $\mathcal{F}$, the trace of the antifield t^* associated with the mixed symmetry $(k,1)$ tensor field, the traces of the antifields denoted by $\mathcal{G}^*$, or the trace of a tensor defined via the antisymmetrized first-order derivatives of the field $(k,1)$ and denoted by F ($\mathcal{F}$ and the traces F, t^*, and $\mathcal{G}^*$ with a single exception related to the last variables are purely antisymmetric tensors of strictly positive form degrees). In strictly positive values of agh appear this time representatives from $H(\delta|d) \setminus H^{\mathrm{inv}}(\delta|d)$ that normally involves fields and/or antifields from both sectors. The functions U and V play the role of parametrizing functions at the level of the first-order deformation describing "effective" couplings in $D = k+3$ in the sense that each piece that collects all the terms depending on a single function of the type U or V is again independently consistent. Since each term from the coupling first-order deformation is a monomial of degree one with respect to all the generators from the $(k,1)$ sector previously mentioned, it can be decomposed in terms of them. We emphasize that the trace F couples with an object denoted by M, which contains only generators from the BF sector and is a form of degree k with components of agh between 0 and k. At this stage we assemble the overall first-order deformation as the sum between that describing the BF self-interactions and the one associated with the "effective" couplings. All the parameterizing functions are still arbitrary. In the next step of the deformation procedure we generate the second-order (in the coupling constant) component. Based on a remarkable property of the antibracket between the coupling first-order deformation with itself and evaluating the remaining antibrackets among the two types of first-order deformations, we conclude that:

A. the existence of the second-order deformation restricts the parameterizing functions to satisfy a set of algebraic and differential equations, to be called also consistency equations;

B. this set is organized into two subsets, among which the former is precisely given by the consistency equations for the pure BF case adapted to $D = k+3$;

C. the latter displays a similar form, namely, each equation reduces to a sum of products between two functions or between a function and the first-order derivative of another one, but now one of these functions is connected to the BF sector and the other to the "effective" couplings;

D. on the non-zero solutions of the overall set of consistency equations (which can be shown to exist) the second-order deformation is non-zero and its non-integrated density follows by contracting the object M (form of degree k with maximum antighost number k) with itself and the deformations of order three and higher can be taken to vanish;

E. a direct consequence of the above statement resides in the development of the second-order deformations as a sum of pieces with agh between 0 and $2k$;

F. since M depends only on the BF generators, it follows that the second-order deformation will participate only in self-interactions of this sector. In addition, the dependency of M only on the functions U, V and on their derivatives will be transferred at the level of the second-order deformation by products between two such functions or their derivatives;

G. surprisingly, the above mentioned BF self-interactions are granted only by the presence of the considered DFLG. Indeed, in the absence of the sector $(k,1)$ the object M vanishes via the vanishing of all U or V functions. Moreover, we have shown that the second-order deformation for a pure BF model is indeed vanishing in any dimension D.

The results exposed so far show that the fully deformed solution of the master equation for a topological BF model with a maximal field spectrum and the DFLG in terms of a tensor field with the mixed symmetry $(k,1)$ in $D=k+3$, with the properties of being non-trivial, in agreement with all the working hypotheses, and consistent to all orders in the coupling constant, reduces to the sum among the solution of the master equation for the starting free models, the first-order deformation with its two components (BF and respectively coupling the two sectors), and the second-order deformation (depending only on the BF BRST generators). Moreover, the parametrizing functions are solutions to the consistency equations. By projecting the overall deformed solution on the various values of agh we are able to identify all the ingredients of the associated interacting theory in $D=k+3$:

I. the Lagrangian action contains interaction vertices of order one and two in λ, those of order one being associated with the self-interactions among the BF fields and also with their couplings to the tensor field $(k,1)$, while those of order two correspond only to BF self-interactions;

II. the Lagrangian action containing the "effective" couplings is of order one in λ, and its vertices may be put into a one-to-one correspondence with the terms of maximum agh, $k+1$, from the associated first-order deformation via the identifications $\mathcal{F} \longleftrightarrow F$, $C \longleftrightarrow B$, $\eta \longleftrightarrow A$ and by maintaining the accompanying U or V functions;

III. the above identification allows one to state that all the coupling vertices are linear in the trace of the tensor F from the $(k,1)$ sector and include, beside the functions denoted by U or V and depending on the BF scalar field, either the form $\overset{[3]}{B}$, or a product between a B-like form (of minimum form degree 4) and at lest one A field, or only A forms (at least two). The first two couplings break the PT invariance, while the last preserves this symmetry;

IV. all the interaction vertices contain the functions Z, W, M, U, or V depending on the scalar field viewed as non-zero solutions to the consistency equations;

V. the gauge transformations of the tensor field with the mixed symmetry $(k,1)$ get modified at order one in λ by terms depending on the BF gauge parameters;

VI. excepting the scalar field and the form $\overset{[1]}{A}$, the gauge symmetries of the remaining BF fields become deformed at order one in λ by two categories of quantities implying the $(k,1)$ sector — some linear in the first-order antisymmetrized space-time derivatives of the gauge parameter $\theta_{(0)}$ and the others linear in the trace of the tensor F (the last terms is forbidden with respect to $\overset{[2]}{A}$);

VII. all the BF fields allowing for gauge transformations of order one in λ linear in F gain also second-order transformations, obtained from the first-order ones by replacing F with the antighost number zero piece of the object M (up to a numerical factor);

VIII. the gauge algebra becomes open, unlike the free limit, and the reducibility relations hold now only on the deformed stationary surface. In addition, there appear non-vanishing structure functions even at order two in λ, and some of the reducibility functions and accompanying coefficients are deformed up to the same order.

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